Considering Passive Joints in Cooperative Manipulation as Functional Redundancy

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Abstract: In this paper, it is studied how a certain class of physical constraints can be represented by a continuous set of orientations and how this representation is especially suitable for representing the constraints imposed by revolute and spherical joints. Further, it is shown how this can be used to represent the freedom of passive joints. The main contribution of the paper is that it is recognised that in cooperative manipulation, the representation developed can be used to show how the introduction of passive joints at the end of a kinematic chain gives the same advantages as functional redundancy for single manipulators. This is used to improve manipulability and performance. Similar to functional redundancy the freedom of the passive joint is task dependent and the type of the passive joint must be chosen with the task in mind. Due to this observation, this paper proposes to consider the last passive joint a part of the tool. The manipulator can then be a standard industrial manipulator with functional redundancy in the specifications of the orientation of the last joint. It is shown that by introducing a passive joint at the end of the manipulator chain the dynamic load carrying capacity is maintained or increased while each manipulator is given a freedom equal to functional redundancy. The workspace of the manipulators is also increased.

1. INTRODUCTION

Passive joints are joints without actuation, so the joint position cannot be controlled directly through the actuator torque of the respective joint. For conventional manipulators these joints are impossible or very hard to control. For cooperative manipulators rigidly connected to a load the positions of the passive joints can be controlled indirectly due to the constraints imposed by the closed chain.

This paper proposes an approach where the passive joint is looked upon as a freedom in the orientation of the link previous to this joint. The joint positions that satisfy the physical constraints imposed by the joint are represented as a continuous set of allowed orientations and can hence be treated similar to functional redundancy in the end-effector specifications. Then, as a part of the path planning algorithm, the optimal position of the passive joint is found from the set of allowed positions of the joint. The optimal joint position is then subject to some optimisation criteria, such as to improve performance, collision avoidance or to improve the workspace of the manipulator.

Functional redundancy is introduced to the system when the dimension of the task space is lower than the dimension of the operational space. For applications such as painting, welding and high pressure steam cleaning, the orientation about the tool axis does not affect the performance and this can be set freely. When the last joint is set passive, this freedom can be utilised in the same way to improve performance. It is shown that for two or more manipulators handling an object, this freedom can be used to improve performance.

There are some examples in literature where passive joints are used at the end of a manipulator chain. In Yeo et al. [1999], a passive joint was introduced at the end of a non-holonomic manipulator to increase the mobility of two cooperating sawing-robots. Another example is the Stewart platform, where only the prismatic joints are actuated and the spherical joints are passive. To the best of the authors' knowledge, the introduction of passive joints at the end of a conventional (industrial) manipulator has not earlier been treated as functional redundancy.

There are several advantages in introducing passive joints at the end of the manipulator chain:

- The weight and size of the manipulator is reduced. This is especially advantageous since the weight is reduced at the end of the manipulator chain.
- If the freedom of the passive joint is treated as functional redundancy, an optimisation algorithm similar to the one introduced in From and Gravdahl [2007b] can be implemented for collision avoidance, increase the workspace or other optimisation criteria. This is possible due to the analytical representation of the passive joint, given by the quaternion volume in From and Gravdahl [2007a].
- Because the passive joint is at the end of the manipulator chain it can be considered a part of the tool so that the rest of the manipulator can be a standard industrial manipulator.
- By considering the passive joint a part of the tool, the axis/axes of the passive joint can be tool dependent and then also task dependent.

- The understanding of passive joints is also important when it comes to understanding and handling free-swinging joint faults (FSJF), see Tinós et al. [2006].

2. REPRESENTING ROTATIONS

Most of the fundamental principles of rotation were presented in two papers by Leonhard Euler in 1775 (Alpern et al. [1993]). The first paper shows that any rotation can be accomplished by a sequence of three rotations about the coordinate axes. In the second paper, Euler states that any orientation can be represented by a rotation of some angle ϕ about some fixed axis $\boldsymbol{n}$.

2.1 The Unit Quaternion

The unit quaternion representation closely relates to the results presented in Euler's second paper. Any positive rotation ϕ about a fixed unit vector $\boldsymbol{n}$ can be represented by the four-tuple (Kuipers [2002])

$$Q = \begin{bmatrix} q_0 \\ \boldsymbol{q} \end{bmatrix}, \tag{1}$$

where $q_0 \in \mathbb{R}$ is known as the scalar part and $\boldsymbol{q} \in \mathbb{R}^3$ as the vector part. $Q(\phi, \boldsymbol{n})$ is written in terms of ϕ and $\boldsymbol{n}$ by

$$q_0 = \cos\left(\frac{\phi}{2}\right), \qquad \boldsymbol{q} = \sin\left(\frac{\phi}{2}\right)\boldsymbol{n}, \tag{2}$$

Hence, Q and $-Q$ represent the same rotation. This is referred to as the dual covering. The quaternion identity is given by $Q_I = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}^{\mathsf{T}}$. A multiplication of two quaternions is given by a quaternion product and is written in vector algebra notations as

$$P * Q = \begin{bmatrix} p_0 q_0 - \boldsymbol{p} \cdot \boldsymbol{q} \\ p_0 \boldsymbol{q} + q_0 \boldsymbol{p} + \boldsymbol{p} \times \boldsymbol{q} \end{bmatrix}. \tag{3}$$

Let $P = \begin{bmatrix} p_0 & p_1 & p_2 & p_3 \end{bmatrix}^{\mathsf{T}}$ and $Q = \begin{bmatrix} q_0 & q_1 & q_2 & q_3 \end{bmatrix}^{\mathsf{T}}$. Then (3) is written as

$$P * Q = \begin{bmatrix} p_0 q_0 - p_1 q_1 - p_2 q_2 - p_3 q_3 \\ p_0 q_1 + p_1 q_0 + p_2 q_3 - p_3 q_2 \\ p_0 q_2 + p_2 q_0 + p_3 q_1 - p_1 q_3 \\ p_0 q_3 + p_3 q_0 + p_1 q_2 - p_2 q_1 \end{bmatrix}. \tag{4}$$

A pure quaternion is a quaternion with zero scalar part. Any vector, $\bar{\boldsymbol{v}} = \begin{bmatrix} x & y & z \end{bmatrix}^{\mathsf{T}}$ can be represented by a pure quaternion $\boldsymbol{v} = \begin{bmatrix} 0 & \bar{\boldsymbol{v}}^{\mathsf{T}} \end{bmatrix}^{\mathsf{T}}$. The conjugate of a quaternion is defined as $Q^* = \begin{bmatrix} q_0 & -q_1 & -q_2 & -q_3 \end{bmatrix}^{\mathsf{T}}$.

2.2 Quaternions and Rotations

Let a vector, $\bar{\boldsymbol{v}}_1$, be represented by the pure quaternion $\boldsymbol{v}_1$. This vector is rotated ϕ radians around the axis $\boldsymbol{n}$ by

$$\boldsymbol{v}_2 = Q * \boldsymbol{v}_1 * Q^*. \tag{5}$$

Every vector $\bar{\boldsymbol{v}} \in \mathbb{R}^3$ can be represented by a pure quaternion. The resulting vector, $\bar{\boldsymbol{v}}_2$, is then of the same length as $\bar{\boldsymbol{v}}_1$ if and only if Q is a unit quaternion. The quaternion representation also leads to a useful formula for finding the shortest rotation from one orientation to another. Let P and Q be two orientations. Then, by taking

$$E = P^* * Q, \tag{6}$$

E will rotate P into Q by the shortest rotation.

Note that equation (6) rotates one frame into another frame. By a *frame* it is meant a coordinate system in $\mathbb{R}^3$ using Cartesian coordinates. One frame with respect to another frame represents three degrees of freedom and is referred to as an *orientation*. Equation (5) rotates one vector into another vector and has two degrees of freedom (e.g. longitude and latitude, see Ahuactzin and Gupka [1999]). A unit vector with respect to a unit reference vector is referred to as a *direction*. Henceforth, when referred to direction, this is the direction of the z-axis of the body frame with respect to the z-axis of the reference frame. This is the axis pointing out of the end-effector and is referred to as the central axis.

3. QUATERNION VOLUMES

3.1 Representation of physical constraints

A joint is a physical constraint that restricts the position and orientation of one link with respect to another. For a revolute joint, this constraint can be represented by a continuous set of orientations where each orientation corresponds to one position of the joint.

In From and Gravdahl [2007a] functional redundancy was represented by a set of allowed orientations of the end effector, represented by the quaternion volume. The restrictions of the end effector were hence given by some performance criteria and the task to be performed, such as the pointing task, orientation error, etc. In this paper a similar approach is used to represent the freedom of one link with respect to another due to a physical constraints.

A systematic approach on how to represent sets of orientations, as presented in From and Gravdahl [2007a], is given.

Definition 3.1. (Quaternion Volume). A quaternion volume, $Q^\otimes$, is defined as

$$Q^\otimes \triangleq \{Q(\phi_1, \ldots, \phi_n, \boldsymbol{n}_1, \ldots, \boldsymbol{n}_n) \mid \phi_{1,min} \le \phi_1 \le \phi_{1,max}$$
$$\vdots \tag{7}$$
$$\phi_{n,min} \le \phi_n \le \phi_{n,max}\}$$

for $n \ge 1$ and where

$$Q(\phi_1, \ldots, \phi_n, \boldsymbol{n}_1, \ldots, \boldsymbol{n}_n) = Q(\phi_1, \boldsymbol{n}_1) * \cdots * Q(\phi_n, \boldsymbol{n}_n). \tag{8}$$

The shape of the quaternion volume is given by the choice of the axes $\boldsymbol{n}_i$ and the size is given by the restriction on ϕ_i. If the axes $\boldsymbol{n}_i$ are chosen as the unitary axes, this will simplify the expressions considerably and a wide range of physical constraints can be represented in this way. As will be clear in the next section, the quaternion volume is also well suited to represent the freedom of joints. The definition given above with $\boldsymbol{n}_i$ chosen to be the unitary axes can then be used to represent, among others, revolute, spherical and universal joints.

3.2 Quaternion Volumes by Rotations Sequences

A revolute joint can be represented simply by an arbitrary rotation about one axis. To represent the freedom of joints where the rotations are not restricted to one axis only, a more general representation of the set of allowed orientations can be given by rotation sequences, see

Kuipers [2002] and From and Gravdahl [2007a]. A rotation sequence describes a rotation about one coordinate axis followed by a rotation about another of the coordinate axes in the rotated coordinate system. A general framework on how to construct easily visualisable quaternion volumes by rotation sequences is presented. The rotation sequence starts with two subsequent rotations about two coordinate axes, represented by the quaternion Q_s. This defines the direction of the z-axis. The last degree of freedom is added by a rotation about the direction vector (central axis), here the z-axis, by Q_z. In equation (5), let Q_z represent the vector to be rotated and Q_s the quaternion describing the direction of this vector. Then the rotation sequence

$$\mathcal{V} = Q_s * Q_z * Q_s^* \qquad (9)$$

represents the direction of the z-axis for a given rotation Q_s given by the direction of the vector part of $\mathcal{V}$ and the rotation about the z-axis given by the scalar part or length of the vector part of $\mathcal{V}$ by $\psi = 2\arcsin(\|\bar{\boldsymbol{v}}\|) = 2\arccos(v_0)\mathrm{sgn}(\psi)$. Henceforth, $\mathcal{V}$ is called a *visualising quaternion*. Note that $\mathcal{V}$ does not represent a rotation. It is used as a tool to visualise rotations and as a help to define an appropriate set of frames for different applications. The visualising quaternion and the corresponding quaternion should be viewed upon as a pair $(Q, \mathcal{V})$, where the visualising quaternion $\mathcal{V}$, gives an intuitive description of a rotation of a frame by Q.

Let the vector part of the visualising quaternion be plotted as a point in the xyz-sphere. Then the direction of the z-axis, rotated by the corresponding quaternion is given by the vector from the origin to this point, and the rotation about the z-axis itself is given by the length of this vector. Hence, a continuous set of quaternions (a quaternion volume) is represented by a "cloud" in the xyz-sphere describing the corresponding set of orientations.

The quaternion that rotates the reference frame into the orientation described by equation (9) is then given by

$$Q = Q_s * Q_z. \qquad (10)$$

Finally, the quaternion volume is given by restricting the allowed rotations of each quaternion.

Given a visualising quaternion volume by the sequence

$$\mathcal{V}^\otimes = Q_s^\otimes * Q_z^\otimes * (Q_s^\otimes)^* \qquad (11)$$

and the restrictions on $Q_s^\otimes$ and $Q_z^\otimes$. Then the corresponding quaternion volume that results in the set of orientations described by $\mathcal{V}^\otimes$ is given by

$$Q^\otimes = Q_s^\otimes * Q_z^\otimes \qquad (12)$$

with the same restrictions applied to $Q^\otimes$ as to $\mathcal{V}^\otimes$.

3.3 Representation of the freedom of passive joints

The freedom introduced by the physical constraints of a revolute or spherical passive joint can be represented by a continuous set of frames. Typical physical constraints, i.e. types of joints, are:

- Revolute joint - rotations about one axis only are allowed (Figure 1).
- Cone-shaped spherical joint - rotations about all axes are allowed, but within certain limits (Figure 2a).

- Pyramid-shaped spherical joint - rotations about two axes allowed, but within certain limits (Figure 2b).

In this paper, the body-fixed x-axis of the end effector is assumed to point in the direction of the gravity force and the z-axis is assumed parallel to the gripper in a right-handed coordinate frame. Only revolute, spherical and universal passive joints are treated. When a joint is considered as a physical constraint, all the joints listed above are easily represented by the set of allowed positions of the joint represented by the quaternion volume.

Example 1. (Passive revolute joint). A passive revolute joint is assumed attached at the end of a manipulator chain. The three cases illustrated in Figure 1 are considered. Assume that the passive joint can rotate about the x-axis as in Figure 1b). If the freedom is restricted to $\phi_{min} < \phi < \phi_{max}$ the freedom can be represented by

$$Q_{free}^\otimes = \begin{bmatrix} \cos\left(\frac{\phi}{2}\right) & \sin\left(\frac{\phi}{2}\right) & 0 & 0 \end{bmatrix}^\mathsf{T}, \text{ for } \phi_{min} < \phi < \phi_{max}. \qquad (13)$$

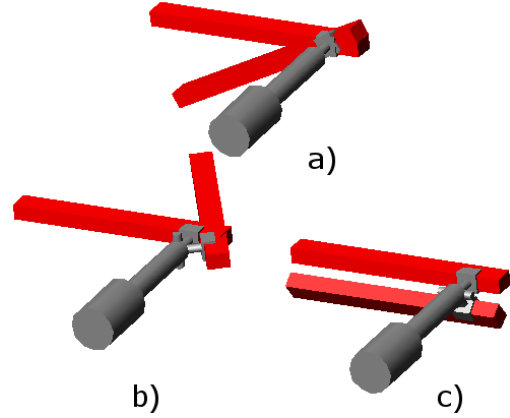Similarly for the y-axis and the z-axis.

Fig. 1. Passive joint about a) the z-axis, b) the x-axis and c) the y-axis for a single manipulator.

Example 2. (Passive spherical joint). The approach presented in Section 3.2 is well suited for representing a passive spherical joint. All the allowed directions that the joint can take are given by

$$Q_s^\otimes(\alpha, \beta) = Q_z^\otimes(\alpha, \boldsymbol{z}) * Q_y^\otimes(\beta, \boldsymbol{y}), \qquad (14)$$

and is represented by

$$Q_s^\otimes(\alpha, \beta) = \begin{bmatrix} \cos\left(\frac{\alpha}{2}\right)\cos\left(\frac{\beta}{2}\right) \\ -\sin\left(\frac{\alpha}{2}\right)\sin\left(\frac{\beta}{2}\right) \\ \cos\left(\frac{\alpha}{2}\right)\sin\left(\frac{\beta}{2}\right) \\ \sin\left(\frac{\alpha}{2}\right)\cos\left(\frac{\beta}{2}\right) \end{bmatrix}, \qquad (15)$$

and some restriction on α and β. If, in addition a rotation about the z-axis is considered, the set of allowed orientations can be visualised by the vector part of

$$\mathcal{V}^\otimes(\alpha, \beta, \gamma) = \begin{bmatrix} \cos(\frac{\gamma}{2}) \\ \sin(\frac{\gamma}{2})\sin(\beta)\cos(\alpha) \\ \sin(\frac{\gamma}{2})\sin(\alpha)\sin(\beta) \\ \sin(\frac{\gamma}{2})\cos(\beta) \end{bmatrix}. \qquad (16)$$

This is illustrated in figure 2a). This set is given by all quaternions that satisfy

$$Q^\otimes = Q_s^\otimes * Q_z^\otimes = \begin{bmatrix} \cos(\frac{\beta}{2})\cos(\frac{\gamma}{2} + \frac{\alpha}{2}) \\ \sin(\frac{\beta}{2})\sin(\frac{\gamma}{2} - \frac{\alpha}{2}) \\ \sin(\frac{\beta}{2})\cos(\frac{\gamma}{2} - \frac{\alpha}{2}) \\ \cos(\frac{\beta}{2})\sin(\frac{\gamma}{2} + \frac{\alpha}{2}) \end{bmatrix} \quad (17)$$

and the restrictions

$$a_{min} \leq \alpha \leq a_{max} \quad (18)$$
$$0 \leq \beta \leq b_{max} \quad (19)$$
$$c_{min} \leq \gamma \leq c_{max} \quad (20)$$

If the joint allows rotations about the x-axis followed by the y-axis, this can be given by a similar quaternion volume (see From and Gravdahl [2007a]). This is illustrated in Figure 2b). The universal joint (U-joint), which consists of two consecutive rotations with orthogonally intersecting axes, is one example of such a joint.
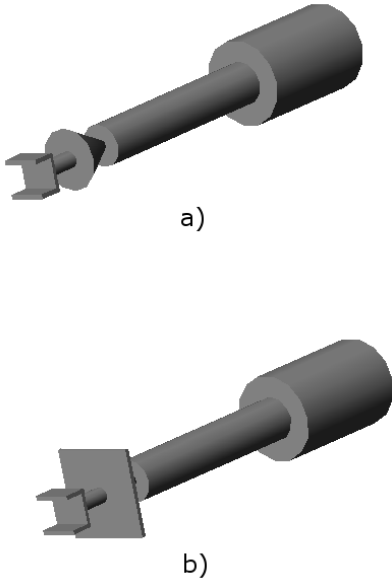

a)

b)

Fig. 2. a) spherical passive joint, b) a passive joint about two axes.

4. COOPERATIVE MANIPULATION

The passive joint can be added to the end of a holonomic manipulator or it can replace the last joint of a holonomic manipulator. In the first case the passive joint can be a part of the tool and its type and axis/axes can be chosen with respect to the tool and task. A standard industrial manipulator can then be used for a wide variety of tasks since the passive joint changes with the task.

In the second case, the last joint of a manipulator is replaced by a passive joint to save weight, size or manufacturing costs. A manipulator is said to be holonomic if its controllable degrees of freedom are equal to or greater than the dimension of the operational space. If one of the actuated joints is replaced by a passive joint, the manipulator may lose this property. It is then said to be non-holonomic. If the last joint of a holonomic manipulator is passive, the manipulator is task dependent and can only be used for one type of tasks.

An important observation, however, is that, in both cases described above, the ability for two or more cooperating manipulators to handle an object is not necessarily reduced. The constraints themselves can introduce redundancy to the system (Murray et al. [1994]), even if each manipulator looked at separately is non-holonomic. Hence, two rigidly connected manipulators with the non-holonomic property due to passive joints can have the holonomic property (Liu et al. [1999]). This makes the introduction of passive joints in cooperative manipulation especially appealing. As long as the number of actuated joints is larger than the dimension of the task space and the geometry of the manipulator is chosen with care, the mobility of the object is not reduced. The constraints imposed by the closed chain allow the object to be rigidly manipulated by cooperative manipulators even if all manipulators have passive joints at the end of the manipulator chain.

It is important that the passive joints are chosen with the task in mind and in such a way that the joint position is not affected by the gravity forces or the motion of the load. Recall that the x-axis of the end effector points in the direction of the gravity force and that the z-axis is parallel to the gripper. Figure 3 shows how a choice of passive joints together with the constraints imposed by the closed chain does not reduce the ability of the manipulators to handle an object. Note also that the position and orientation of the object is not affected by the position of the passive joint. This freedom can hence be utilised for collision avoidance, to increase the workspace or to improve performance in general.
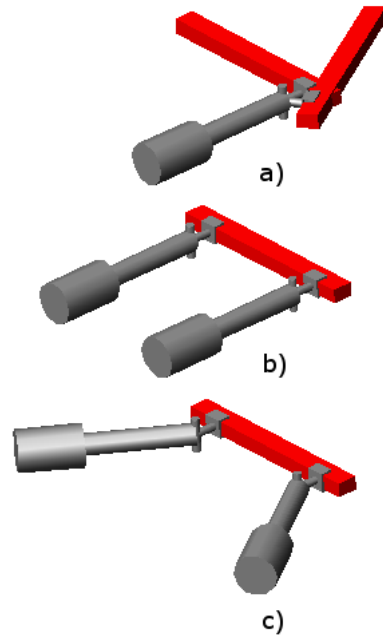

a)

b)

c)

Fig. 3. a) A single manipulator with a passive joint, b) two manipulators rigidly connected, c) the passive joint treated as functional redundancy. All passive joints rotate about the x-axis.

Example 3. Two manipulators moving a beam
Assume two manipulators moving a beam, both connected to the same side of the beam. Then, assume a gripper with a passive joint that can rotate about the x- or z-axis. This will not reduce the ability of the two manipulators to move the beam. For the x-axis, this is illustrated in Figure 3c).

Example 4. If the passive joint rotates about the y-axis, two manipulators can handle the beam if they are attached one on each side. This is because in this case, the passive joints don't rotate about the same axis, but about two parallel axes. Then the gravity force will not affect the configuration of the object. This is a general result.

4.1 Dynamic Load Carrying Capacity

The dynamic load carrying capacity (DLCC) is a measure of how much one or more manipulators can lift for a specific trajectory (Wang and Ravani [1988]). It is a useful tool to compare manipulators with different geometry. In this paper, the DLCC is used to compare manipulators with different passive joints with standard manipulators.

The DLCC algorithm finds the maximum possible weight of the load for each point on a trajectory. The maximum weight the manipulator(s) can handle is then given by the minimum of these. This paper proposes a modification of this algorithm. When the last joint is passive, this freedom is exploited and for each point on the trajectory, the position of the passive joint that results in the highest weight of the load is found. In this way it is possible to compare different types of passive joints with conventional manipulators.

4.2 Grasping

For the manipulator to be able to grasp an object, some kind of control of the passive joint is needed. One simple approach is a locking mechanism which locks the joint in a given position so that the object can be grasped. If more mobility is needed, an actuator can be mounted on the joint. If this is used only during the grasping and not during manipulation of the object, this can be made smaller and lighter than conventional actuators.

5. NUMERICAL RESULTS

5.1 Workspace

Two manipulators were to lift a beam, following the path of a half-circle in the yz-plane. As a measurement of how the workspace of the manipulators changes when passive joints are introduced at the end of the kinematic chain the maximum radius of the half circle which the beam can follow is found. Table 5.1 shows how the maximum radius of the circle changes for the task and different types of joints described in Examples 2, 3 and 4. Two types of spherical joints were considered, one which allows an orientation of $20°$ with respect to the previous joint, and one which allows $40°$. It is difficult to find a general approach to how the workspace of the manipulator changes with the introduction of a passive joint because the type of passive joint is task-dependent. It also depends on the tool and the type of the last actuated joint. The tool used

in this paper was a 20cm long gripper with an orientation of 30 degrees about about the y-axis with respect to the last joint. The position of the passive joint was found by a simple geometric analysis based on the approach presented in From and Gravdahl [2007b], and is not an optimal solution to the problem. This approach is well suited for real-time applications. The manipulator used in the all the simulations is the ABB IRB-5400.

Passive Joint	Increase in workspace [%]
Fixed	0
x-axis	4.5
y-axis	6.3
z-axis	10.8
Spherical $20°$	6.9
Spherical $40°$	12.3

Table 1. The workspace for different passive joints

5.2 Performance

Two manipulators were set to take an object along a path (the half-circle in the yz-plane). As a mean to measure the performance, the torques needed to perform the task are calculated.
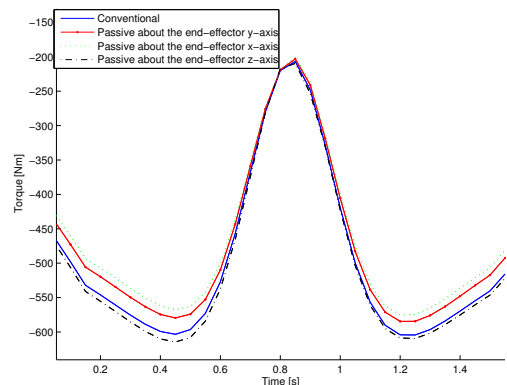

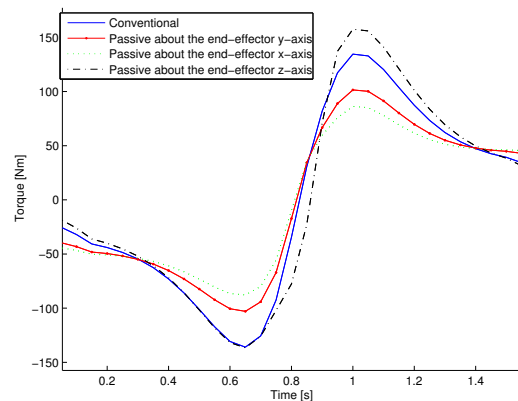Fig. 4. Torques of joint 3 for different types of passive jonts.

Fig. 5. Torques of joint 4 for different types of passive jonts.

When the passive joint is a revolute joint, the torques of joins 3 and 4 are shown in Figures 6 and 7, respectively.

The torques for the other joints are approximately the same for all the approaches. Note that a passive joint that rotates about the z-axis gives the biggest workspace, but also requires most torque to follow the given trajectory.

When the passive joint is a sperical joint with maximum orientation of $20°$, the torques of joints 1 and 2 are found in Figures 6 and 7. Joints 1 and 2 are shown because these are the critical joints when it comes to the torque limits.
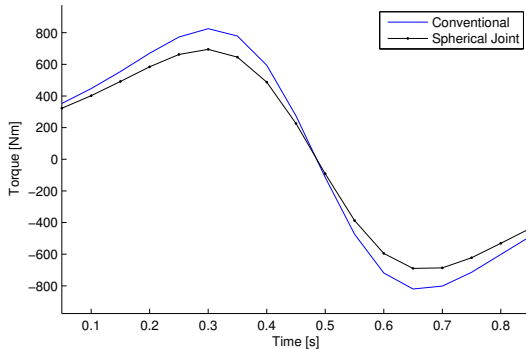

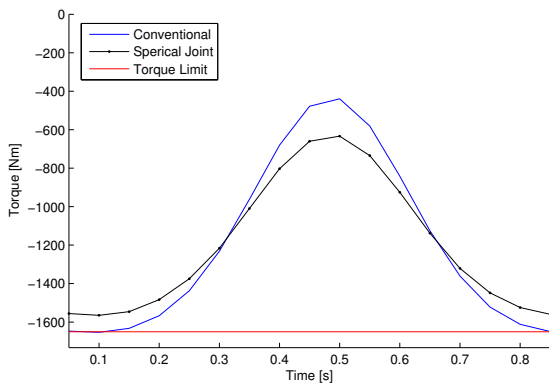Fig. 6. Torque of joint 1 with a spherical passive joint.

Fig. 7. Torque of joint 2 with a spherical passive joint.

5.3 Dynamic Load Carrying Capacity

The dynamic load carrying capacity (DLCC) was found for each of the five types of passive joints. The manipulators were to follow a half circle in the yz-plane with an average velocity of $1m/s$. The maximum weight for each case is found in table 5.3.

Passive Joints	Maximum weight [kg]
Fixed	75
x-axis	91
y-axis	95
z-axis	89
Spherical 20°	80
Spherical 40°	87

Table 2. DLCC for different passive joints

6. CONCLUSION

By introducing a passive joint at the end of the manipulator chain in cooperative manipulation, each manipulator can be treated as if functional redundancy were present. The freedom introduced by the passive joint can be exploited to improve performance in terms of lower torques and joint velocities. It is also shown that the workspace of the manipulators can be increased and that the dynamic load carrying capacity in each case increases. Each application must be considered separately and the passive joint must be chosen with the task in mind. This is important to ensure mobility of the object. What type of passive joint to choose also depends on the performance criteria to be improved.

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