Two General State Feedback Control Laws for Compressor Surge Stabilization*

Nur Uddin and Jan Tommy Gravdahl

***Abstract*— Active surge control system (ASCS) can be classified into two types: upstream energy injection and downstream energy dissipation [1]. Two novel state feedback control laws termed ϕ-control for the upstream energy injection and ψ-control the downstream energy dissipation are presented. Both state feedback control laws are derived by using the Lyapunov based control method such that the closed loop systems are global asymptotic stable (GAS). The ϕ-control applies feedback from the compressor mass flow sensor to generate extra pressure to the compressor upstream line, while the ψ-control generates an extra flow out of the plenum using feedback from the compressor discharged pressure and the plenum pressure. Both state feedback control laws offer a minimum number of sensors requirement. Moreover, the ψ-control requires feedback from pressure sensors only which are readily available and make real-time implementation of the system to be easier.**

I. INTRODUCTION

The operating area of a compressor is commonly shown by a compressor map. The map shows curves of compressor produced pressure versus compressor mass flow. Figure 1 shows an example of compressor map for a constant compressor speed. The compressor operating area at lower mass flow is limited by a surge line and stone wall at the higher mass flow. Surge line is a limit of stability where the right side area of the line is stable while the left side area is unstable. Operating the compressor in the unstable area result in compressor surge, while operating a compressor beyond the stonewall result in compressor choke [2].

Compressor choke is a condition when the gas velocity relative to the blade is equal to the speed of sound such that the compressor cannot pump any more gas [3]. According to [4], a compressor can only operate in stonewall if the head (energy) required by the process system is low enough such that the compressor operates in the high-flow-rate region of the compressor map. The high flow rate requires a larger pipe diameter, which is more costly. For economic reasons, this is a rare case because the process pipe is designed to minimize the pipe diameter and therefore increase the head required at high flow rates. Stonewall is also not a destructive phenomenon. Therefore, surge receives more attention in compressor studies. Moreover, the high-efficiency compressor operating points are located close to the surge line.

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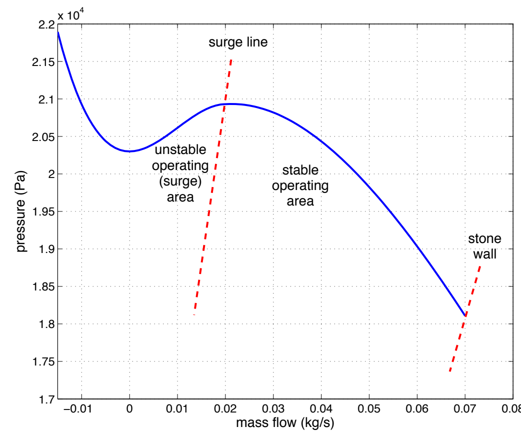

Fig. 1. A compressor map with surge line.

Compressor surge is an aerodynamic instability in the compression system and results in an axisymmetric oscillation of the compressor flow and the compressor produced pressure. It is also defined as a condition where the pressure developed by a compressor (upstream pressure) is less than the system pressure (downstream pressure) [3]. It occurs due to inability of the impeller to produce the amount of required energy for the process system [4]. Surge is physically indicated by pressure fluctuation, reversal of flow, temperature fluctuation and followed by severe vibration. Surge leads to compressor damage especially at the rotating parts, for examples: compressor blades, shaft and bearing, and also pipeline connections and structure. Therefore, a compressor is always prevented to enter surge during the operation. There are two methods to prevent a compressor from experiencing surge: surge avoidance system (SAS) and active surge control system (ASCS).

SAS is the traditional method to prevent a compressor entering surge. SAS works by defining a surge control line which is located on the right side of the surge line as a limit of the minimum compressor flow. It makes the compressor operating point not reach the surge line. The margin between surge control line and surge line is known as surge margin. The surge margin is defined by a compressor operator and 10 % is commonly used. The SAS method is implemented through a recycling flow mechanism by using a recycle valve and recycle line or blowing-off flow mechanism by using a blow-off valve. When the compressor operating point is crossing the surge control line, the recycle valve or the blow-off valve will open to discharge the downstream fluid and result in increasing the flow such that the compressor operating point will stay at the surge control line. This method works well to prevent a compressor from entering surge and is commonly applied in industrial compressors. However, applying SAS reduces the compressor operating envelope as the limit of minimum compressor flow is surge control line instead of surge line. Moreover, the compressor operating points with high efficiency are commonly located closed to the surge line such that a compressor equipped with SAS may not be able to reach the highest efficiency.

As opposed to the SAS which limits the compressor operating area by a surge control line, ASCS is stabilizing surge by an active element such that the compressor operating point is allowed to cross the surge line into the stabilized surge area such that the operating envelope is enlarged. The active element, or actuator, is driven by a controller based on a state feedback control law. The ASCS method was introduced by [5]. ASCS is offering an enlargement of the compressor operating envelope towards lower flows and an opportunity to reach higher compressor efficiency. Several studies by applying different control methods and different actuators have been presented. Nonlinearity in the compressor dynamics gives a challenge in designing the surge control. Several studies by applying linear and non-linear control methods to design ASCS controller have been presented. The linear control design is done by linearizing the compressor dynamics at an operating point to get a linear compressor model such that linear control methods can be applied. Examples of ASCS designed by using linear control theory have been presented in [6]–[8]. The linear control design is able to stabilize compressor surge, but it only achieves local asymptotic stability with limited region of attraction (operating area). On the other hand, nonlinear control methods are promising global asymptotic stability (GAS). The GAS is proved by the Lyapunov stability method which is requiring a Lyapunov function. Backstepping is a systematic non-linear control method to find a state feedback and Lyapunov function such that GAS of the closed loop system is guaranteed. Several works on applying Lyapunov-based or backstepping methods for active surge control were presented in [9]–[11]. However, the method may result in a complicated and impractical state feedback [11].

A compressor model is required in surge control design. Greitzer compression model [12] is one of the most applied models in surge control studies. The Greitzer model is able to predict the transient response subsequent to a perturbation from steady operating condition.

Several actuators have been applied in active surge control and example includes: moveable plenum wall [6], close coupled valve [9], drive torque [13], magnetic bearing [14] and piston actuation [11]. Consult [15] for more actuators applied in ASCS. Considering the working principle of the actuators in stabilizing surge, ASCS can be classified into two types: upstream energy injection and downstream energy dissipation [1]. The upstream energy injection is increasing the pressure at compressor upstream line while

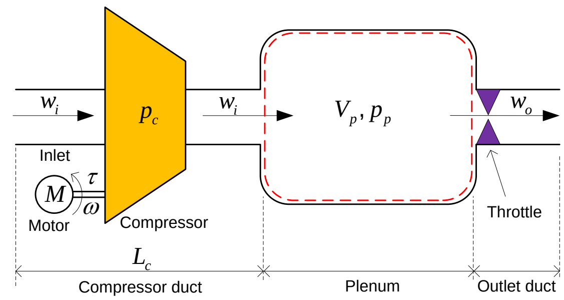

Fig. 2. Model of single compression system.

the downstream energy dissipation is decreasing the pressure at the compressor downstream line (plenum) by flowing out more fluid from the plenum.

This paper presents two new general state feedback controls called ϕ-control and ψ-control for compressor surge stabilization by using ASCS in the types of upstream energy injection and downstream energy dissipation, respectively. Both state feedback controls are derived by using Lyapunov based control method such that the GAS of the closed loop system is guaranteed.

II. Compression System

The Greitzer compression model is shown in Figure 2 and the dynamic equations are given as follows [12]:

$$\dot{w}_i = \frac{A_c}{L_c}(p_c - p_p) \tag{1}$$

$$\dot{p}_p = \frac{a_0^2}{V_p}(w_i - w_o), \tag{2}$$

where w_i is the inlet mass flow, A_c is the compressor duct cross-sectional area, L_c is the effective length of the equivalent compressor duct, p_c is the compressor pressure rise, p_p is the plenum pressure, a_0 is the speed of sound, V_p is the plenum volume, and w_o is the outlet mass flow. It is assumed that the pressures are measured relative to the ambient pressure.

The outlet mass flow is the set point which represents the desired mass flow of a compressor operation. The inlet mass flow will be equal to the outlet mass flow at steady state. The compressor operating point is changing by adjusting the outlet mass flow, where it is physically done by adjusting a throttle. The outlet mass flow is defined by:

$$w_o = k_T u_T \sqrt{p_p} \tag{3}$$

where k_T is the throttle constant and u_T is the throttle opening with range value from 0 to 100 %.

Defining constants $B_1 = \frac{A_c}{L_c}$ and $B_2 = \frac{a_0^2}{V_p}$, the dynamic equations can be expressed as:

$$\dot{w}_i = B_1(p_c - p_p) \tag{4}$$

$$\dot{p}_p = B_2(w_i - w_o). \tag{5}$$

The compressor pressure rise is plotted as a function of the flow for several compressor speeds in a compressor map and is commonly provided by the compressor manufacturer. The map can be obtained by collecting data in a compressor performance test and approximated by a mathematical function. Approximation by a cubic function for constant compressor speed was introduced by [16] as follows:

$$p_c(w_i) = p_{c_0} + H\left[1 + \frac{3}{2}\left(\frac{w_i}{W} - 1\right) - \frac{1}{2}\left(\frac{w_i}{W} - 1\right)^3\right], \quad (6)$$

where p_{c_0} is the shut-off value of the axisymmetric characteristic, W is the semi-width of the cubic axisymetric compressor characteristic, and H is the semi-height of the cubic axisymetric compressor characteristic; consult [16] for more detailed definitions.

III. Principal Work of Active Surge Control

Modeling a compressor system by using bond graph was presented in [1]. Analysis of the bond graph model results in two basic surge solutions for stabilizing compressor surge: upstream energy injection and downstream energy dissipation. Several presented actuators for ASCS are basically working based on one of the two approaches. Based on the Greitzer model, the upstream energy injection results in additional upstream pressure and the downstream energy dissipation results in additional flow out from the plenum in order to stabilize compressor surge. Derivations of state feedback control law for the both basic surge solutions by using Lyapunov control method are presented as follow.

A. Upstream energy injection

A bond graph model of a compression system with active surge control system in the class of upstream energy injection is shown Figure 3. Dynamic equations of the system based on the bond graph model are given as follows[1]:

$$\begin{aligned} I\dot{q}_i &= p_c - p_p + e_{19} & (7)\\ C\dot{p}_p &= q_i - q_o, & (8) \end{aligned}$$

where $I = \frac{\rho L_c}{A_c}$, $C = \frac{V_p}{\rho a_0^2}$, e_{19} is an effort (pressure p_u) generated by an active element S_u, and q is volumetric flow. The relation with mass flow is $w = \rho q$. By assuming the flow is incompressible and using the defined constants B_1 and B_2, the dynamic equations (7) and (8) can be expressed as follows:

$$\begin{aligned} \dot{w}_i &= B_1(p_c - p_p + p_u) & (9)\\ \dot{p}_p &= B_2(w_i - w_o). & (10) \end{aligned}$$

Theorem 1: A ϕ-control with a state feedback $p_u = -k_1(w_i - w_{i_{ref}})$ where $k_1 > k_m$ and $k_m = \left.\frac{\partial p_c}{\partial w_i}\right|_{\max}$, makes the operating point of a compressor system in (9) and (10) at mass flow $w_{i_{ref}}$ to be GAS.

Proof: Define new variables $z_1 = p_c - p_p + p_u$ and $z_2 = w_i - w_o$, and a Lyapunov function candidate

$$V_1 = \frac{B_1}{2}z_1^2 + \frac{B_2}{2}z_2^2. \quad (11)$$

[1]Notation according to [1]

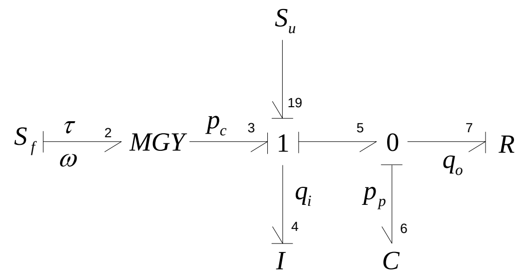

Fig. 3. Bond graph model of active surge control in a class of upstream energy injection [1].

The time derivative of V_1 is given as follows:

$$\begin{aligned} \dot{V}_1 &= B_1 z_1 \dot{z}_1 + B_2 z_2 \dot{z}_2\\ &= B_1(p_c - p_p + p_u)(\dot{p}_c - \dot{p}_p + \dot{p}_u) & (12)\\ &\quad + B_2(w_i - w_o)(\dot{w}_i - \dot{w}_o)\\ &= \dot{w}_i(\dot{p}_c - \dot{p}_p + \dot{p}_u) + \dot{p}_p\dot{w}_i - \dot{p}_p\dot{w}_o\\ &= \dot{w}_i\dot{p}_c - \dot{w}_i\dot{p}_p + \dot{w}_i\dot{p}_u + \dot{p}_p\dot{w}_i - \dot{p}_p\dot{w}_o\\ &= \frac{\partial p_c}{\partial w_i}\dot{w}_i^2 + \dot{w}_i\dot{p}_u - \frac{\partial w_o}{\partial p_p}\dot{p}_p^2 \end{aligned}$$

Let $\dot{p}_u = -k_1\dot{w}_i$,

$$\begin{aligned} \dot{V}_1 &= \left(\frac{\partial p_c}{\partial w_i} - k_1\right)\dot{w}_i^2 - \frac{\partial w_o}{\partial p_p}\dot{p}_p^2\\ &= \left(\frac{\partial p_c}{\partial w_i} - k_1\right)B_1^2(p_c - p_p + p_u)^2 - \frac{\partial w_o}{\partial p_p}B_2^2(w_i - w_o)^2\\ &= \left(\frac{\partial p_c}{\partial w_i} - k_1\right)B_1^2 z_1^2 - \frac{\partial w_o}{\partial p_p}B_2^2 z_2^2, & (13) \end{aligned}$$

where $\frac{\partial w_o}{\partial p_p} = \frac{w_o}{2p_p} > 0$, see the Appendix for the derivation of $\frac{\partial w_o}{\partial p_p}$. Selecting $k_1 > k_m$ where $k_m = \left.\frac{\partial p_c}{\partial w_i}\right|_{\max}$ results in $\dot{V}_2 < 0$ such that the closed loop system of (9) and (10) is GAS. The state feedback control is given as

$$p_u = -k_1\int \dot{w}_i dt = -k_1(w_i - w_{i_{ref}}), \quad (14)$$

where $w_{i_{ref}}$ is the desired compressor mass flow which is the desired throttle mass flow w_o. ■

The value of k_m is exist and bounded, where an example is shown in Figure 1. By applying (6) to approximate a compressor performance curve, a calculation results in $k_m = \frac{3H}{2W}$. Equation (14) is the state feedback control to stabilize surge by maintaining the compressor to operate at the desired mass flow and called as ϕ-control.

B. Downstream energy dissipation

A bond graph model of a compression system with active surge control system in the class of downstream energy dissipation is shown in Figure 4. Dynamic equation of the system based on the bond graph model is given as follows[2]:

$$\begin{aligned} I\dot{q}_i &= p_c - p_p & (15)\\ C\dot{p}_p &= q_i - q_o - f_{20}, & (16) \end{aligned}$$

[2]Notation according to [1]

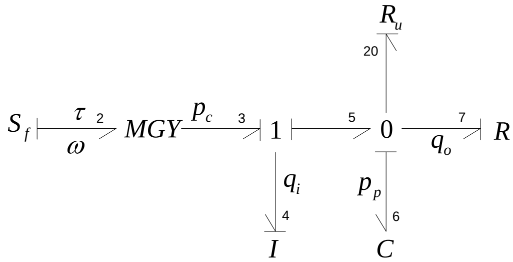

Fig. 4. Bond graph model of active surge control in a class of downstream energy dissipation [1].

where f_{20} is a flow out from the plenum generated by an active element R_u and it is defined as the volumetric flow q_u. By assuming the flow is incompressible and using the defined constants B_1 and B_2, the dynamic equations (15) and (16) can be expressed as:

$$\begin{aligned} \dot{w}_i &= B_1(p_c - p_p) && (17)\\ \dot{p}_p &= B_2(w_i - w_o - w_u). && (18) \end{aligned}$$

Theorem 2: A ψ-control with a state feedback $w_u = -\frac{k_2 B_1}{B_2}(p_c - p_p)$, where $k_m < k_2 < k_n$ with $k_m = \frac{\partial p_c}{\partial w_i}\Big|_{\max}$ and $k_n = \frac{\partial p_p}{\partial w_o}\Big|_{\min}$, makes the operating point of a compression system in (17) and (18) at a desired mass flow w_o to be GAS.

Proof: Recall the defined variables $\theta_1 = p_c - p_p$ and $\theta_2 = w_i - w_o$, and define a Lyapunov function candidate:

$$V_2 = \frac{B_1}{2}\theta_1^2 + \frac{B_2}{2}\theta_2^2. \qquad (19)$$

Time derivative of V_2 is given as follows:

$$\begin{aligned} \dot{V}_2 &= B_1\theta_1\dot{\theta}_1 + B_2\theta_2\dot{\theta}_2\\ &= B_1(p_c - p_p)(\dot{p}_c - \dot{p}_p) + B_2(w_i - w_o)(\dot{w}_i - \dot{w}_o)\\ &= \dot{w}_i(\dot{p}_c - \dot{p}_p) + B_2(w_i - w_o - w_u + w_u)(\dot{w}_i - \dot{w}_o)\\ &= \dot{w}_i\dot{p}_c - \dot{w}_i\dot{p}_p + B_2(w_i - w_o - w_u)(\dot{w}_i - \dot{w}_o)\\ &\quad + B_2 w_u(\dot{w}_i - \dot{w}_o)\\ &= \dot{w}_i\dot{p}_c - \dot{w}_i\dot{p}_p + \dot{p}_p(\dot{w}_i - \dot{w}_o) + B_2 w_u(\dot{w}_i - \dot{w}_o)\\ &= \dot{w}_i\dot{p}_c - \dot{w}_i\dot{p}_p + \dot{p}_p\dot{w}_i - \dot{p}_p\dot{w}_o + B_2 w_u(\dot{w}_i - \dot{w}_o)\\ &= \dot{w}_i\dot{p}_c - \dot{p}_p\dot{w}_o + B_2 w_u\dot{w}_i - B_2 w_u\dot{w}_o\\ &= \frac{\partial p_c}{\partial w_i}\dot{w}_i^2 + B_2 w_u\dot{w}_i - B_2 w_u\dot{w}_o - \dot{p}_p\dot{w}_o. \qquad (20) \end{aligned}$$

Let $w_u = -\frac{k_2}{B_2}\dot{w}_i$ such that

$$\dot{V}_2 = \left(\frac{\partial p_c}{\partial w_i} - k_2\right)\dot{w}_i^2 + k_2\dot{w}_i\dot{w}_o - \dot{p}_p\dot{w}_o,$$

and by applying Young's inequality:

$$\begin{aligned} \dot{V}_2 &\leq \left(\frac{\partial p_c}{\partial w_i} - k_2\right)\dot{w}_i^2 + k_2\left(\frac{\dot{w}_i^2}{2} + \frac{\dot{w}_o^2}{2}\right) - \dot{p}_p\dot{w}_o\\ &= \left(\frac{\partial p_c}{\partial w_i} - \frac{k_2}{2}\right)\dot{w}_i^2 + \frac{k_2}{2}\dot{w}_o^2 - \dot{p}_p\dot{w}_o\\ &= \left(\frac{\partial p_c}{\partial w_i} - \frac{k_2}{2}\right)\dot{w}_i^2 + \frac{k_2}{2}\left(\frac{\partial w_o}{\partial p_p}\dot{p}_p\right)^2 - \dot{p}_p\left(\frac{\partial w_o}{\partial p_p}\dot{p}_p\right)\\ &= \left(\frac{\partial p_c}{\partial w_i} - \frac{k_2}{2}\right)\dot{w}_i^2 + \left[\frac{k_2}{2}\left(\frac{\partial w_o}{\partial p_p}\right)^2 - \frac{\partial w_o}{\partial p_p}\right]\dot{p}_p^2\\ &= \left(\frac{\partial p_c}{\partial w_i} - \frac{k_2}{2}\right)\dot{w}_i^2 + \left[\frac{k_2}{2}\left(\frac{\partial w_o}{\partial p_p}\right)^2 - \frac{\left(\frac{\partial w_o}{\partial p_p}\right)^2}{\frac{\partial w_o}{\partial p_p}}\right]\dot{p}_p^2\\ &= \left(\frac{\partial p_c}{\partial w_i} - \frac{k_2}{2}\right)\dot{w}_i^2 + \left(\frac{k_2}{2} - \frac{\partial p_p}{\partial w_o}\right)\left(\frac{\partial w_o}{\partial p_p}\right)^2\dot{p}_p^2\\ &= \left(\frac{\partial p_c}{\partial w_i} - \frac{k_2}{2}\right)B_1^2(p_c - p_p)^2\\ &\quad + \left(\frac{k_2}{2} - \frac{\partial p_p}{\partial w_o}\right)\left(\frac{\partial w_o}{\partial p_p}\right)^2 B_2^2(w_i - w_o - w_u)^2\\ &= \left(\frac{\partial p_c}{\partial w_i} - \frac{k_2}{2}\right)B_1^2\theta_1^2\\ &\quad + \left(\frac{k_2}{2} - \frac{\partial p_p}{\partial w_o}\right)\left[\frac{\partial w_o}{\partial p_p}B_2\left(\theta_2 + \frac{k_2}{B_2}B_1\theta_1\right)\right]^2. \qquad (21) \end{aligned}$$

Selecting $k_m < \frac{k_2}{2} < k_n$, where $k_m = \frac{\partial p_c}{\partial w_i}\Big|_{\max}$ and $k_n = \frac{\partial p_p}{\partial w_o}\Big|_{\min}$, results in $\dot{V}_2 < 0$ such that the closed loop system in (17) and (18) is GAS by a state feedback control

$$w_u = -\frac{k_2 B_1}{B_2}(p_c - p_p). \qquad (22)$$

■

It is shown in Appendix that $\frac{\partial p_p}{\partial w_o} = \frac{2p_p}{w_o}$ and therefore $k_n = \frac{2p_p}{w_o}\Big|_{\min}$. The value of k_n can be obtained by finding the operating point at the maximum mass flow in the compressor map for the corresponding compressor speed. Equation (22) is the state feedback control to stabilize surge by maintaining the compressor discharged pressure to be equal to the plenum pressure and we call it as ψ-control. Note that the (22) requires feedback from pressure only, a quantity that is more readily available for measurement than mass flow. It makes real-time implementation of the system to be easier.

C. Actuator

Block diagrams of ASCS in the class of upstream energy injection and downstream energy dissipation are shown in Figures 5 and 6, respectively. Actuator is the mean to execute the state feedback control into physical action. The type of actuator is not limited as long as it can generate mass flow w_u or pressure p_u. The actuator should have fast response and accurate value. However, an ideal actuator may not available such that it will be a trade-off between actuator and the surge control performance.

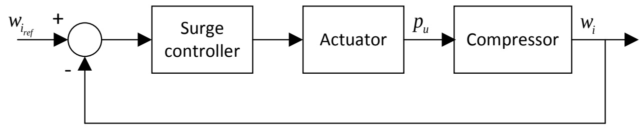

Fig. 5. Block diagram of active surge control with upstream energy injection (ϕ-control).

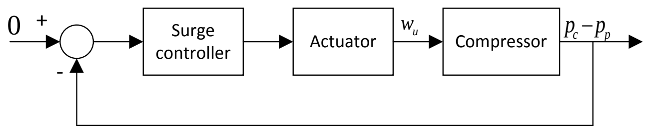

Fig. 6. Block diagram of active surge control with downstream energy dissipation (ψ-control).

TABLE I
SIMULATION PARAMETERS

Parameter	Value	Unit	Parameter	Value	Unit
a_0	340	m/s	V_p	0.1	m^3
L_c	0.41	m	A_c	0.0038	m^2
p_{c0}	20.3	kPa	H	0.315	kPa
W	0.01	kg/s			

IV. SIMULATION

Simulations are presented to demonstrate the performance of the ASCS in the class of upstream energy injection and downstream energy dissipation for stabilizing compressor surge. The simulations use parameters given in Table I and a compressor map in Figure 1. The compressor map shows that the surge point, which is the minimum stable compressor mass flow, is located at mass flow 0.02 kg/s and pressure 20.93 kPa, while the choke point, which is the maximum compressor mass flow, is located at mass flow 0.07 kg/s and pressure 18.1 kPa. Based on the available data, we obtain $k_m = 4.725 \times 10^4$ kPa.s/kg, $k_n = 5.17 \times 10^5$ kPa.s/kg, $B_1 = 9.3 \times 10^{-3}$ m, and $B_2 = 1.156 \times 10^6$ $m^{-1}s^{-2}$. Therefore, the control parameters for the upstream energy injection (ϕ-control) is $k_1 > 4.725 \times 10^4$ and the downstream energy dissipation (ψ-control) is $4.725 \times 10^4 < \frac{k_2}{2} < 5.17 \times 10^5$. In this simulation, we choose $k_1 = 6.24 \times 10^4$ and $k_2 = 10^5$.

Two simulations will be carried out to show the performance of the the ϕ-control and the ψ-control in stabilizing surge and the simulation scenario is give as follows. Both simulations will be done by operating the compressor at point A where the mass flow is 0.06 kg/s as the initial point and at $t = 40$ seconds the operating point is shifted to B where the mass flow is 0.01 kg/s by closing the throttle. The operating points A is located in the stable operating area while B is in the surge area. The compressor will enter surge and the surge control will be activated a few second after the compressor surge. The simulation results are given as follows.

The performance of the upstream energy injection (ϕ-control) ASCS is shown in Figure 7 for the time response of the system and Figure 8 for the system trajectory. The compressor operating point is shifted from point A to B by reducing the mass flow from 0.06 to 0.01 kg/s at $t = 40$. The compressor enters surge after the the operating point passed the surge point as shown in compressor operating trajectory in Figure 8.The surge is shown by oscillations at compressor mass flow oscilation (w_i) and plenum pressure (p_p) in Figure 7 and limit cycle in Figure 8. The surge is stabilized after the surge control is activated at $t = 43$ seconds and the system is finally operating stable at point B where the mass flow is 0.01 kg/s.

The performance of the downstream energy dissipation (ψ-control) ASCS is shown in Figure 9 for the time responses and Figure 10 for the system trajectory. Figure 9 is shown that the compressor enters to surge at $t = 40$ seconds after the operating point is shifted to point B. The surge is stabilized after the surge control is activated at $t = 43.6$ seconds and the system is operating stable at point B where the mass flow is 0.01 kg/s. The reason of activating the surge control at $t = 43.6$ seconds and not at $t = 43$ seconds as in previous simulation is solely to give more clear visualization of the compressor operating trajectory in Figure 10.

Figure 9 also shows that the control mass flow w_u instantaneously jumped when the surge control was activated. It requires the actuator to generated a high mass flow instantaneously and is not practical. The jump of w_u is the effect of derivative control where the w_u is proportional to $\dot{w}_i$ and it can be eliminated by limiting w_u in a reasonable value. The w_u should not more than w_i. Figure 12 shows simulation result of applying limiter to the w_u with limitation range $-0.02 \leq w_u \leq 0.02$ kg/s. The surge is stabilized although the w_u is limited. A limiter is not necessary for the upstream energy injection as it is not a derivative control. The p_u is proportional to the deviation of w_i to $w_{i_{ref}}$.

V. CONCLUSION

Two state feedback controls called ϕ-control for ASCS with upstream energy injection and ψ-control for ASCS with downstream energy dissipation were presented including the GAS proof of the closed loop systems. Both state feedback controls stabilize surge and the closed loop system is GAS. An advantage of the proposed schemes is the reduced sensor requirements, ϕ-control is only requiring a mass flow sensor and the ψ-control is only requiring two pressure sensors to measure compressor discharge pressure and the plenum pressure.

The ψ-control can be directly implemented in real-time system as the pressure sensor is readily available, and an example of applying the ψ-control including experimental test results have been presented in [17]. However, it is not the same for the ϕ-control due to the available mass flow sensor has slow response and not able to capture compressor surge. As an alternative mass flow observer introduced in [18], [19] can be used to get an estimated mass flow. Application of the both feedback controls depends on the actuator type for ASCS, whether it is in the class of upstream energy injection or downstream energy dissipation.

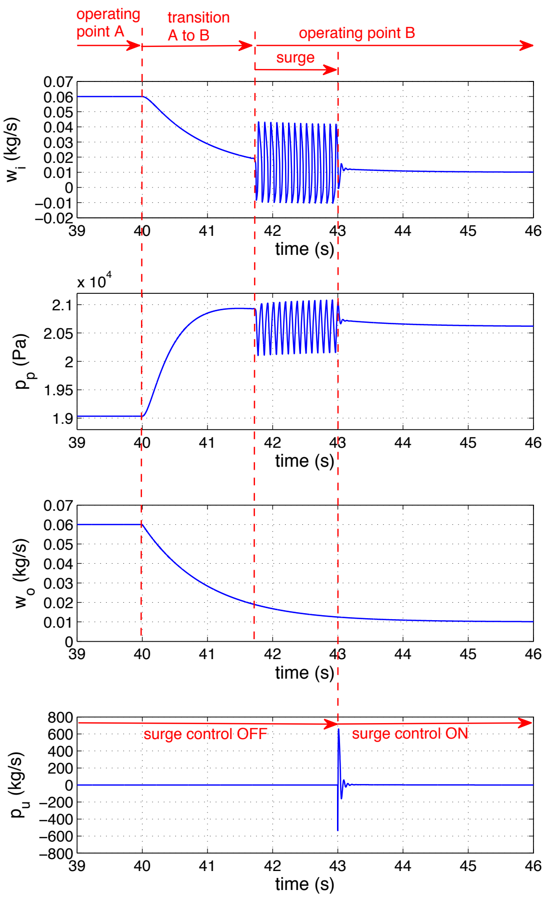

Fig. 7. Time responses of a compression system equipped by an active surge control using upstream energy injection.

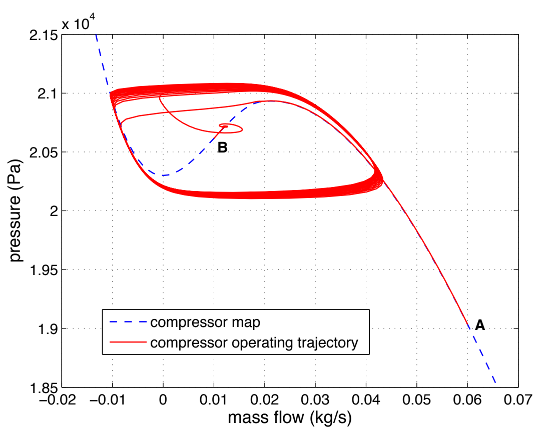

Fig. 8. Operating trajectory of a compression system equipped by an active surge control using upstream energy injection.

APPENDIX

The derivative of outlet mass flow with respect to plenum pressure is given as follows:

$$w_o = k_T u_T \sqrt{p_p}$$

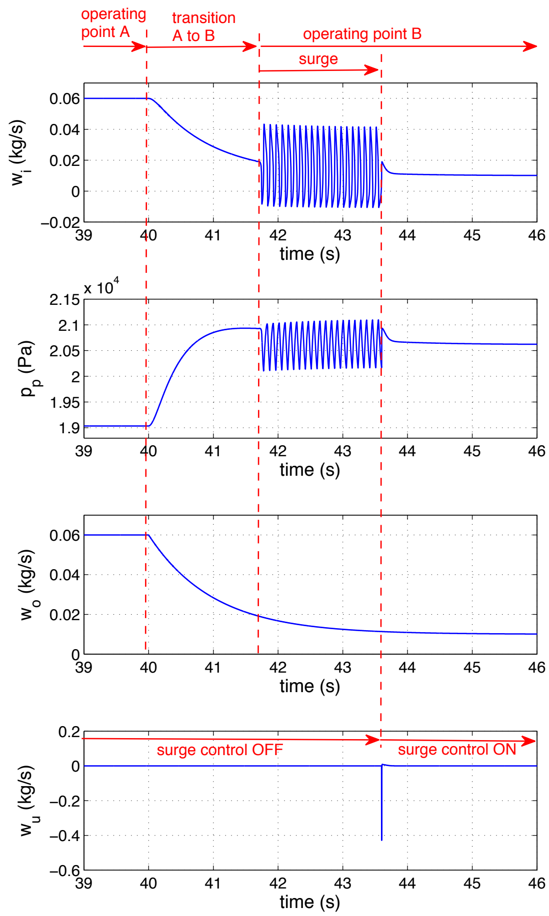

Fig. 9. Time responses of a compression system equipped by an active surge control using downstream energy dissipation.

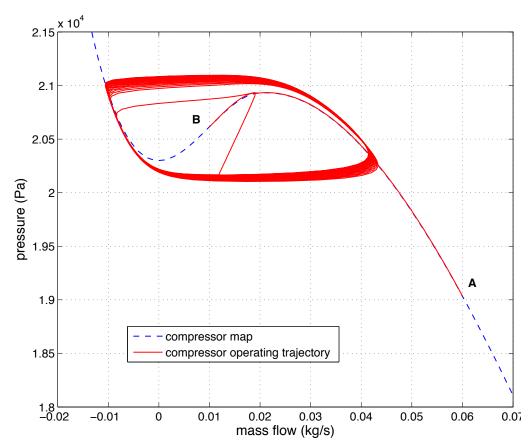

Fig. 10. Operating trajectory of a compression system equipped by an active surge control using downstream energy dissipation.

$$\frac{dw_o}{dp_p} = \frac{k_T u_T}{2\sqrt{p_p}} = \frac{w_o}{2p_p}, \tag{23}$$

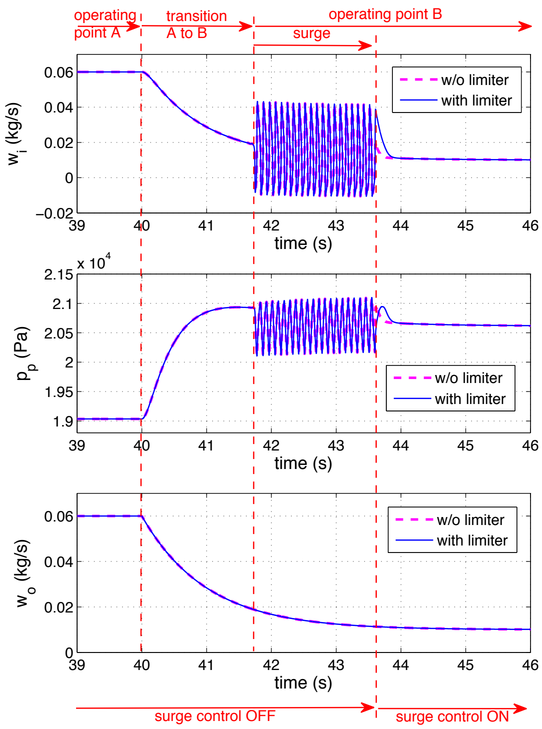

Fig. 11. Time responses of a compression system equipped by an active surge control using downstream energy dissipation including limiter for the actuator.

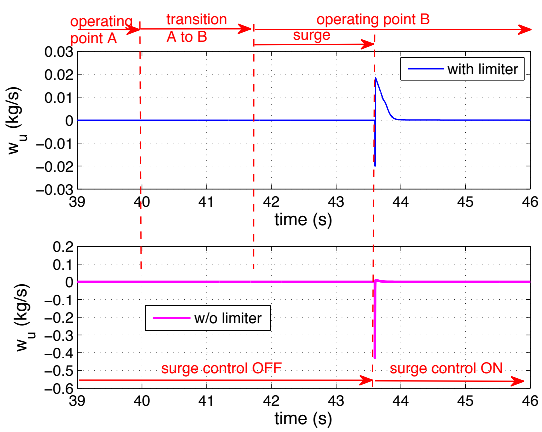

Fig. 12. Comparison of control mass flow used in downstream energy dissipation with limiter and without limiter.

and the derivative of outlet mass flow with respect to plenum pressure

$$p_p = \frac{1}{(k_T u_T)^2} w_o^2$$

$$\frac{dp_p}{dw_o} = \frac{2}{(k_T u_T)^2} w_o = \frac{2p_p}{w_o}.$$

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