

# On active surge control of compressors using a mass flow observer

Bjørnar Bøhagen and Jan Tommy Gravdahl

Department of Engineering Cybernetics, NTNU, N-7491 Trondheim, Norway

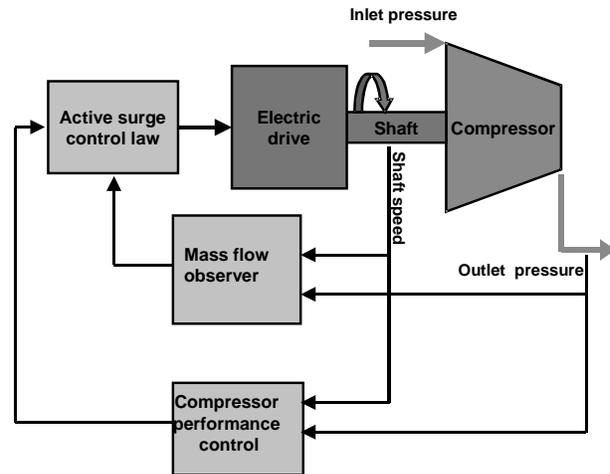
## Abstract

A globally exponentially stable (GES) observer for the mass flow in a compression system is proposed. Further, a previously proposed active surge control scheme is shown to be GES. This nonlinear control scheme employs the drive torque of the compressors drive unit as the control variable in an active surge control system. The controller is based on feedback from mass flow. It is desirable to avoid this measurement as it is both inaccurate and expensive. Using a nonlinear separation principle, the total system is shown to be globally asymptotically stable (GAS). The results are supported by simulations.

## 1 Background

Towards low mass flows, the stable operating region of centrifugal compressors is bounded due to the occurrence of surge. Surge is an unstable operation mode of the compressor and the stability boundary in the compressor map is called the surge line. Surge is characterized by oscillations in pressure rise and mass flow. These oscillations can cause severe damage to the machine due to vibrations and high thermal loading resulting from lowered efficiency. Traditionally, surge has been avoided using surge avoidance schemes. Such schemes use various measures to keep the operating point of the compressor away from the surge line. Typically, a surge control line is drawn at a distance from the surge line, and the surge avoidance scheme ensures that the operating point does not cross this line. This method restricts the operating range of the machine, and efficiency is limited. Usually a recycle line around the compressor is used as actuation. Active surge control is fundamentally different to surge avoidance in that the unstable phenomenon is sought to be stabilized instead of avoided. Thus the operating regime of the compressor is enlarged.

Active surge control of compressors was first introduced by [1], and since then a number of results have been published. Different actuators have been used, and examples include recycle, bleed and throttle valves, gas injection, variable guide vanes and a number of others. For an overview, consult [2] or [3]. In this work we will



**Figure 1:** The compression system considered consists of a centrifugal compressor driven by an electrical motor and the control system consists of three parts: the active surge control law, the mass flow observer and the performance control.

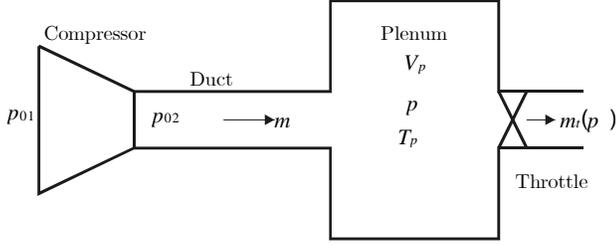
use the approach of [4] and [5] where it was proposed to use the drive unit for active surge control as depicted in Figure 1. The advantage of this approach is that the drive is already present, and no additional actuation device is required. This means that the compressor can be operated at a low flow without recycling, and there is a potential for reduced energy consumption of the compressor.

The controller of [4] was based on feedback from rotational speed and mass flow. It is known that real time measurement of mass flow is both expensive and hampered with high noise levels. For this reason we propose a nonlinear observer for estimation of the mass flow. This estimate is used for feedback to the active surge controller.

## 2 Model

### 2.1 Dynamics

A classical result in the field of compressor surge modeling is the model of Greitzer [6] who modelled a basic compression system consisting of a compressor, a



**Figure 2:** The compressor, plenum, throttle system of [6]

plenum volume, a throttle valve and in-between ducting as shown in Figure 2. In order to study the drive torque as control variable for surge control, we need a model that takes variable speed into account. In [7], the Greitzer-model was further developed, and rotational speed was included as a state in the model. A similar model was derived in [8], using an approach based on energy based analysis. In this paper we will employ the compressor model derived in [8]. The model is derived by calculating the mass balance of the plenum volume, integrating the one dimensional Euler equation (the momentum balance) over the length of the exit duct, and calculating the torque balance of the rotating shaft. The model is written

$$\dot{p} = \frac{a_{01}^2}{V_p}(m - m_t), \quad (1a)$$

$$\dot{m} = \frac{A_1}{L_c}(\Psi_c(m, \omega)p_{01} - p), \quad (1b)$$

$$\dot{\omega} = \frac{1}{J}(\tau_d - \tau_c), \quad (1c)$$

where  $p$  is the plenum pressure,  $m$  is the compressor mass flow,  $\omega$  is the rotational velocity of the shaft,  $\Psi_c(m, \omega)$  is the compressor characteristic,  $m_t$  is the throttle flow,  $A_1$  is the throughflow area,  $L_c$  is the duct length,  $V_p$  is the plenum volume,  $p_{01}$  is the ambient pressure,  $a_{01}$  is the sonic velocity at ambient conditions,  $J$  is the inertia of all rotating parts, and  $\tau_d$  and  $\tau_c$  is the drive torque and compressor load torque, respectively. The throttle flow is given by  $m_t = k_t \sqrt{p - p_{01}}$ , where  $k_t > 0$  is a parameter proportional to throttle opening. The compressor torque  $\tau_c$  is calculated as  $\tau_c = |m|r_2^2\sigma\omega$ , where  $r_2$  is the impeller diameter and  $\sigma$  is the slip factor. The drive torque  $\tau_d$  will be used as the control variable. For a detailed derivation of the model, consult [8].

### 3 Control

In [4] it was shown that previous unstable operating points to the left of the surge line can be made GES by using the rotational speed  $\omega$  of the drive as control variable. A velocity control scheme for the drive torque  $\tau_d$  was also proposed. This scheme guaranteed

exponential convergence to a region around the desired operating point. We will extend these results by proving that the desired operating point can be made GES also when using the drive torque  $\tau_d$  as control. The equilibrium values representing the desired operating point are denoted by  $(\cdot)_0$ , while deviations from the equilibrium are denoted by  $(\bar{\cdot})$ .

**Definition 1** (Deviations from equilibrium)

$$\bar{m} = m - m_0 \quad (2a)$$

$$\bar{p} = p - p_0 \quad (2b)$$

$$\bar{\omega} = \omega - \omega_0 \quad (2c)$$

$$\bar{m}_t = k_t \sqrt{p - p_{01}} - k_t \sqrt{p_0 - p_{01}} = m_t - m_{t0} \quad (2d)$$

$$\bar{\Psi}_c = \Psi_c(m, \omega) - \Psi_c(m_0, \omega_0) = \Psi_c - \Psi_{c0} \quad (2e)$$

$$\bar{\tau}_c = |m|r_2^2\sigma\omega - |m_0|r_2^2\sigma\omega_0 = \tau_c - \tau_{c0} \quad (2f)$$

$$\bar{\tau}_d = \tau_d - \tau_{d0}. \quad (2g)$$

The model (1) in new coordinates  $(\bar{\cdot})$  is written

$$\dot{\bar{p}} = \frac{a_{01}^2}{V_p}(\bar{m} - \bar{m}_t), \quad (3a)$$

$$\dot{\bar{m}} = \frac{A_1}{L_c}(\bar{\Psi}_c p_{01} - \bar{p}), \quad (3b)$$

$$\dot{\bar{\omega}} = \frac{1}{J}(\bar{\tau}_d - \bar{\tau}_c), \quad (3c)$$

where the equilibrium values satisfies  $m_0 = m_{t0}$ ,  $p_0 = \Psi_{c0}p_{01}$  and  $\tau_{d0} = \tau_{c0}$ . In [4] it was shown that when using the control law

$$\bar{\omega} = -c_1 \bar{m} \quad (4)$$

the equilibrium of (3a)-(3b) is made GES. The Lyapunov function

$$V = \frac{V_p}{2a_{01}^2}\bar{p}^2 + \frac{L_c}{2A_1}\bar{m}^2 \quad (5)$$

was used and it was shown that the time derivative of (5) along the solutions of (3a)-(3b) can be upper bounded as

$$\dot{V}(\bar{p}, \bar{m}) < -k_p \bar{p}^2 - k_m \bar{m}^2, \quad (6)$$

where  $k_p > 0$  depends on the slope of the throttle characteristic and  $k_m > 0$  depends on the slope of the compressor characteristic. In (4) the shaft speed is used as control variable. This approach requires a velocity controller for the drive in order to achieve and maintain the speed described by (4), without corrupting the stability guaranteed by this control law. From (4) it can be recognized that the desired shaft speed is given by

$$\omega_d = \omega_0 - c_1 \bar{m}.$$

In [4] the velocity control law

$$\bar{\tau}_d = K_1(\omega_d - \omega) = -K_1 \bar{\omega} - K_1 c_1 \bar{m} = -K_1 \bar{\omega} - K_2 \bar{m},$$

where

$$K_2 = K_1 c_1$$

was proposed and exponential convergence was proved.

**Proposition 1** (*GES equilibrium*)

The dynamics (3) in closed loop with the surge control law

$$\bar{\tau}_d = -K_1\bar{\omega} - K_2\bar{m} \quad (7)$$

where the gain  $c_1$  is chosen according to

$$c_1 > \sup \left( \frac{\partial \Psi_c / \partial m}{\partial \Psi_c / \partial \omega} \right) \quad (8)$$

and the gain  $K_2$  is chosen according to

$$K_2 = K_1 c_1 \quad (9)$$

makes the origin of (3) globally exponentially stable

**Proof:** Consider the Lyapunov function candidate

$$V_1(\bar{p}, \bar{m}, \bar{\omega}) = \frac{V_p}{2a_{01}^2} \bar{p}^2 + \frac{L_c}{2A_1} \bar{m}^2 + \frac{J}{2} \bar{\omega}^2. \quad (10)$$

Using (5) and (6), the time derivative of (10) along the solutions of (3) can be written

$$\begin{aligned} \dot{V}_1 &< -k_p \bar{p}^2 - k_m \bar{m}^2 + J \bar{\omega} \dot{\bar{\omega}} \\ &< -k_p \bar{p}^2 - k_m \bar{m}^2 + \bar{\omega} (\bar{\tau}_d - \bar{\tau}_c) \\ &< -k_p \bar{p}^2 - k_m \bar{m}^2 + \bar{\omega} (-K_1 \bar{\omega} - K_2 \bar{m}) - \bar{\omega} \bar{\tau}_c \\ &< -k_p \bar{p}^2 - k_m \bar{m}^2 - K_1 \bar{\omega}^2 - K_2 \bar{m} \bar{\omega} - \bar{\omega} \bar{\tau}_c. \end{aligned} \quad (11)$$

The compressor load torque and the shaft rotational velocity are assumed to be a passive pair such that  $P_c = \bar{\omega} \bar{\tau}_c \geq 0$ , where  $P_c$  is the power consumed by the compressor load. Further, the  $\bar{m} \bar{\omega}$ -term in (11) can be upper bounded using Young's inequality

$$-K_2 \bar{m} \bar{\omega} \leq \frac{K_2}{\eta_1} \bar{m}^2 + K_2 \eta_1 \bar{\omega}^2, \quad (12)$$

where  $\eta_1 > 0$ . An upper bound on (11) can now be found as

$$\dot{V}_1 < -k_p \bar{p}^2 - \left( k_m - \frac{K_2}{\eta_1} \right) \bar{m}^2 - (K_1 - K_2 \eta_1) \bar{\omega}^2.$$

Choosing  $K_1$ ,  $K_2$  and  $\eta_1$  such that the inequalities

$$\begin{aligned} k_1 &< \frac{2k_p V_p}{a_{01}^2} \\ k_1 &< \frac{2 \left( k_m - \frac{K_2}{\eta_1} \right) L_c}{A_1} \\ k_1 &< 2(K_1 - K_2 \eta_1) J \end{aligned}$$

holds for  $k_1 > 0$ , makes the origin of (3) globally exponentially stable. That is,  $V_1(\bar{p}, \bar{m}, \bar{\omega})$  is positive definite and radially unbounded and its time derivative along the solutions of (3) satisfies

$$\dot{V}_1(\bar{p}, \bar{m}, \bar{\omega}) < -k_1 V_1(\bar{p}, \bar{m}, \bar{\omega}).$$

■

**4 Observer**

Due to the practical difficulties of implementing a controller that depends on feedback from mass flow, we now propose a GES observer for mass flow. The estimated state is denoted by  $(\hat{\cdot})$ , while estimation error is denoted by  $(\tilde{\cdot})$

**Definition 2** (Estimation error)

$$\begin{aligned} \tilde{m} &= m - \hat{m} \\ \tilde{\Psi}_c &= \Psi_c - \hat{\Psi}_c = \Psi_c(m, \omega) - \Psi_c(\hat{m}, \omega). \end{aligned}$$

The observer dynamics are constructed by copying (1b) and are given by

$$\dot{\hat{m}} = \frac{A_1}{L_c} \left( \hat{\Psi}_c p_{01} - p \right) + K_{\tilde{m}} \tilde{m}, \quad (14)$$

where  $K_{\tilde{m}} = \frac{A_1}{L_c} c_2$  is the observer gain, and  $c_2$  is a tuning parameter for the observer. Notice that (14) depends on the unmeasured state  $m$ . Following [9] this is handled by introducing the new variable

$$z = \hat{m} - K_z p,$$

where  $K_z = \frac{V_p}{a_{01}^2} \frac{A_1}{L_c} c_2$ . The observer can now be implemented as

$$\dot{z} = \frac{A_1}{L_c} \left( \hat{\Psi}_c p_{01} - p - c_2 \hat{m} - c_2 m_t \right), \quad (15a)$$

$$\hat{m} = z + K_z p, \quad (15b)$$

where it can be seen that the observer uses measurements of  $\omega$  and  $p$  only. The observer error dynamics are now given by

$$\begin{aligned} \dot{\tilde{m}} &= \dot{m} - \dot{\hat{m}} \\ &= \frac{A_1}{L_c} (\Psi_c p_{01} - p) - \frac{A_1}{L_c} \left( \hat{\Psi}_c p_{01} - p \right) - \frac{A_1}{L_c} c_2 \tilde{m} \\ &= \frac{A_1}{L_c} \left( \tilde{\Psi}_c p_{01} - c_2 \tilde{m} \right). \end{aligned} \quad (16)$$

**Proposition 2** (*GES observer*)

The observer error dynamics

$$\dot{\tilde{m}} = \frac{A_1}{L_c} \left( \tilde{\Psi}_c p_{01} - c_2 \tilde{m} \right)$$

is GES if the observer gain,  $c_2$  is chosen according to

$$c_2 > 2p_{01} \sup \left\{ \frac{\partial \Psi_c}{\partial m} \right\} + \delta_2, \quad (17)$$

where  $\delta_2 > 0$ .

**Proof:** Consider the Lyapunov function candidate

$$V_2(\tilde{m}) = \frac{L_c}{2A_1} \tilde{m}^2 \quad (18)$$

The time derivative of (18) along the solution of (16) is

$$\dot{V}_2(\tilde{m}) = \tilde{m} \left( \tilde{\Psi}_c p_{01} - c_2 \tilde{m} \right) = \tilde{m} \alpha(\tilde{m}) \quad (19)$$

where  $\alpha(\tilde{m}) = \tilde{\Psi}_c p_{01} - c_2 \tilde{m}$ . In order to prove stability, we now have to show that  $\tilde{m} \alpha(\tilde{m}) < 0$  for all  $\tilde{m} \neq 0$ . If it can be proven that  $\alpha(\tilde{m})$  is located in the 2nd and 4th quadrant of the  $(\tilde{m}, \alpha(\tilde{m}))$ -coordinate system, then  $\tilde{m} \alpha(\tilde{m}) < 0$  since  $\tilde{m}$  is located in the 1st and 3rd quadrant. As  $\alpha(0) = 0$ , where it has been used that  $\tilde{\Psi}_c|_{\tilde{m}=0} = 0$ , a sufficient condition for  $\alpha(\tilde{m})$  to be located in the 2nd and 4th quadrant is that  $\alpha(\tilde{m})$  is monotonically decreasing in  $\tilde{m}$ , that is

$$\frac{\partial \alpha(\tilde{m})}{\partial \tilde{m}} < 0. \quad (20)$$

Calculating the partial derivative

$$\begin{aligned} \frac{\partial \alpha(\tilde{m})}{\partial \tilde{m}} &= \frac{\partial \tilde{\Psi}_c}{\partial \tilde{m}} p_{01} - c_2 \frac{\partial \tilde{m}}{\partial \tilde{m}} \\ &= \frac{\partial \tilde{\Psi}_c}{\partial \tilde{m}} p_{01} - \frac{\partial \hat{\Psi}_c}{\partial \tilde{m}} p_{01} - c_2 \\ &= \frac{\partial \tilde{\Psi}_c}{\partial m} \frac{\partial m}{\partial \tilde{m}} p_{01} - \frac{\partial \hat{\Psi}_c}{\partial \tilde{m}} \frac{\partial \hat{m}}{\partial \tilde{m}} p_{01} - c_2 \\ &= \frac{\partial \tilde{\Psi}_c}{\partial m} p_{01} + \frac{\partial \hat{\Psi}_c}{\partial \tilde{m}} p_{01} - c_2 \end{aligned}$$

it can be recognized that (20) is satisfied if  $c_2$  is chosen according to

$$c_2 > \sup \left\{ \frac{\partial \tilde{\Psi}_c}{\partial m} + \frac{\partial \hat{\Psi}_c}{\partial \tilde{m}} \right\} p_{01} + \delta_2 = 2p_{01} \sup \left\{ \frac{\partial \tilde{\Psi}_c}{\partial m} \right\} + \delta_2, \quad (21)$$

where  $\delta_2 > 0$ . Using (21) we can now write (19) as

$$\dot{V}_2(\tilde{m}) < -\delta_2 \tilde{m}^2 < -2\delta_2 \frac{A_1}{L_c} V_2(\tilde{m}),$$

which implies that the origin of (16) is globally exponentially stable. That is  $V_2(\tilde{m})$  is positive definite and radially unbound and the time derivative along the solutions of (16) can be written

$$\dot{V}_2(\tilde{m}) < -k_2 V_2(\tilde{m}),$$

where  $k_2 = 2\delta_2 \frac{A_1}{L_c}$ . ■

**Remark 1** *It is interesting to notice that the analysis that follows from using the stabilizing term  $-c_2 \tilde{m}^2$  in (19) is equivalent to the stability analysis when using a closed coupled valve (CCV) for active surge control. This can e.g. be seen in [2].*

## 5 Stability of overall system

In this section we investigate the stability of the overall system when the estimate,  $\hat{m}$ , of the mass flow,  $m$ , is

used in the feedback controller (7). To prove stability of the cascaded system, we will use the methodology of [10] which allows for tuning of the controller and the estimator separately. In [10] stability conditions for non-autonomous nonlinear systems in cascade are given. Our approach is inspired by [11]. For stability results for autonomous nonlinear cascaded systems using passivity properties, the reader is referred to [12].

**Theorem 3** (*GAS Output Feedback*)  
The controller

$$\bar{\tau}_d = -K_1 \bar{\omega} - K_2 (\hat{m} - m_0) \quad (22)$$

where the controller gains,  $K_1$  and  $K_2$ , are chosen according to Proposition 1, the estimate,  $\hat{m}$ , is implemented according to (15) and the observer gain,  $c_2$ , is chosen according to Proposition 2, makes the origin of (3) GAS.

**Proof:** Theorem 3 will be proved by showing that the cascaded system satisfies the assumption of Theorem 2 in [10]. Following the notations of [10] we consider the cascade

$$\begin{aligned} \Sigma_1 &: \dot{x}_1 = f_1(x_1) + g(x)x_2 \\ \Sigma_2 &: \dot{x}_2 = f_2(x_2) \end{aligned}$$

where  $\Sigma_1$  represents (3) using the control law (22),  $\Sigma_2$  represents the observer error dynamics in Proposition 2 with  $c_2$  chosen according to (17), and

$$\begin{aligned} x_1 &= \begin{bmatrix} \bar{p} \\ \bar{m} \\ \bar{\omega} \end{bmatrix}, \\ x_2 &= \tilde{m}, \\ f_1(x_1) &= \begin{bmatrix} \frac{a_{01}^2}{V_p} (\bar{m} - \bar{m}_t) \\ \frac{A_1}{L_c} (\tilde{\Psi}_c p_{01} - \bar{p}) \\ \frac{1}{J} (-K_1 \bar{\omega} - K_2 \bar{m} - \bar{\tau}_c) \end{bmatrix}, \\ f_2(x_2) &= \frac{A_1}{L_c} \left( \tilde{\Psi}_c p_{01} - (c_2 + \delta_2) \tilde{m} \right), \\ g(x) &= \begin{bmatrix} 0 \\ 0 \\ \frac{1}{J} K_2 \end{bmatrix}, \\ V_1(\bar{p}, \bar{m}, \bar{\omega}) &= \frac{V_p}{2a_{01}^2} \bar{p}^2 + \frac{L_c}{2A_1} \bar{m}^2 + \frac{J}{2} \bar{\omega}^2, \end{aligned}$$

where  $V_1$  is positive definite and proper Lyapunov function for  $\dot{x}_1 = f_1(x_1)$ . The Lyapunov function,  $V_1$ , may be written

$$\begin{aligned} V_1(x_1) &= \frac{V_p}{2a_{01}^2} \bar{p}^2 + \frac{L_c}{2A_1} \bar{m}^2 + \frac{J}{2} \bar{\omega}^2 \\ &= \frac{1}{2} x_1^T K x_1 \end{aligned}$$

where  $K = \text{diag} \left\{ \frac{V_p}{a_{01}^2}, \frac{L_c}{A_1}, J \right\}$ . The property

$$\begin{aligned} \left\| \frac{\partial V_1}{\partial x_1} \right\|_2 \|x_1\|_2 &= \|Kx_1\|_2 \|x_1\|_2 \leq \|K\|_2 \|x_1\|_2^2 \\ &\leq \max \left\{ \left( \frac{V_p}{a_{01}^2} \right)^2, \left( \frac{L_c}{A_1} \right)^2, J^2 \right\} \|x_1\|_2^2 \\ &\leq \gamma_1 V_1 \end{aligned}$$

where

$$\gamma_1 \geq 2 \frac{\max \left\{ \left( \frac{V_p}{a_{01}^2} \right)^2, \left( \frac{L_c}{A_1} \right)^2, J^2 \right\}}{\min \left\{ \frac{V_p}{a_{01}^2}, \frac{L_c}{A_1}, J \right\}}$$

together with the property

$$\begin{aligned} \left\| \frac{\partial V_1}{\partial x_1} \right\|_2 &= \|Kx_1\|_2 \leq \|K\|_2 \|x_1\|_2 \\ &\leq \max \left\{ \left( \frac{V_p}{a_{01}^2} \right)^2, \left( \frac{L_c}{A_1} \right)^2, J^2 \right\} \|x_1\|_2 \\ &\leq \max \left\{ \left( \frac{V_p}{a_{01}^2} \right)^2, \left( \frac{L_c}{A_1} \right)^2, J^2 \right\} \mu \\ &\leq \gamma_2 \end{aligned}$$

where  $\gamma_2 = \max \left\{ \left( \frac{V_p}{a_{01}^2} \right)^2, \left( \frac{L_c}{A_1} \right)^2, J^2 \right\} \mu$  and the condition  $\|x_1\|_2 \leq \mu$  has been used, satisfies Assumption 1 in [10]. Investigating  $g(x)$  we find

$$\|g(x)\| = \left( \frac{1}{J} K_2 \right)^2 \leq \theta_1 (\|x_2\|) + \theta_1 (\|x_2\|) |x_1|$$

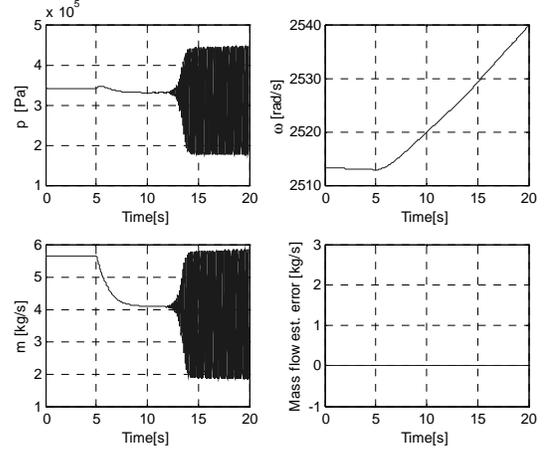
where  $\theta_1 (\|x_2\|) \geq \left( \frac{1}{J} K_2 \right)^2$  and  $\theta_1 (\|x_2\|) = 0$ , and Assumption 2 of [10] is satisfied. The observer error dynamics are globally exponentially stable, that is there exist two positive constants  $\lambda_1$  and  $\lambda_2$ , such that  $\|x_2(t)\| \leq \lambda_1 \|x_2(t_0)\| e^{-\lambda_2(t-t_0)}$ . Integrating this solution over time, we get

$$\begin{aligned} \int_{t_0}^{\infty} \|x_2(t)\| dt &\leq \lambda_1 \|x_2(t_0)\| \int_{t_0}^{\infty} e^{-\lambda_2(t-t_0)} dt \\ &\leq \lambda_1 \|x_2(t_0)\| e^{\lambda_2 t_0} \int_{t_0}^{\infty} e^{-\lambda_2 t} dt \\ &\leq \frac{\lambda_1}{\lambda_2} \|x_2(t_0)\| e^{\lambda_2 t_0} \\ &\leq \phi (\|x_2(t_0)\|) \end{aligned}$$

where  $\phi (\|x_2(t_0)\|) = \frac{\lambda_1}{\lambda_2} \|x_2(t_0)\| e^{\lambda_2 t_0}$  is a class  $\mathcal{K}$  function. Having satisfied Assumptions 1, 2 and 3 and GES of (3), Theorem 2 in [10] states that the cascade is globally asymptotically stable. ■

## 6 Simulations

The model used for simulation is the same as the one used in [4]. The compressor is initially operating in a



**Figure 3:** Compression system driven into surge

stable operating point. After  $t = 5\text{s}$ , a throttle change introduces a drop in mass flow driving the compressor over the surge line.

### 6.1 Surge

In this section it is illustrated that the model is capable of simulating surge. A constant drive torque,  $\tau_d = 400\text{Nm}$ , is applied to the motor, and as can be seen from Figure 3 the system is driven into surge. As can be seen from Figure 3, the estimation error quickly converges to zero.

### 6.2 Simulation of active surge stabilization using feedback from estimated mass flow

In this section the proposed controller (7) using the proposed mass flow observer (15) is simulated. From Figure 4 it can be seen that the system remain stable after the throttle change. The desired shaft speed was chosen to be  $\omega_0 = 400[1/s] \approx 2513.3[\text{rad/s}]$ , controller parameters used were  $c_1 = 100$ ,  $K_1 = 10000$ ,  $K_2 = 500$  and the observer gain is set at  $c_2 = 40000$ . Notice that according to (9),  $K_2 = c_1 K_1$ . Using this large gain, the generated drive torque becomes very large. In the simulations we have therefore chosen the much lower gain  $K_2 = 500$ .

### 6.3 Simulation of active surge stabilization with measurement noise and estimated mass flow

In a real compression system there will be disturbances and measurement noise. In this section noise in the pressure measurement, which is one of the inputs to the observer, is added. By this we wish to illustrate the ability of the observer to reject such disturbances. The measurement noise is zero mean white noise with an amplitude of  $5000\text{Pa}$ . The controller and observer parameters are the same as above. As can be seen from Figure 6, the system remains stable

## 7 Conclusion

We have shown that surge in a centrifugal compression system can be actively stabilized using the drive torque as control variable. The controller uses feedback from the rotational velocity and estimated mass flow. The mass flow observer is based on pressure measurement.

## References

- [1] A. Epstein, J. F. Williams, and E. Greitzer, "Active suppression of aerodynamic instabilities in turbomachines," *Journal of Propulsion and Power*, vol. 5, no. 2, pp. 204–211, 1989.
- [2] J. Gravdahl and O. Egeland, *Compressor surge and rotating stall: modeling and control*. Advances in Industrial Control, London: Springer-Verlag, 1999.
- [3] F. Willems and B. de Jager, "Modeling and control of compressor flow instabilities," *IEEE Control systems*, pp. 8–18, October 1999.
- [4] J. Gravdahl, O. Egeland, and S. Vatland, "Active surge control of centrifugal compressors using drive torque," in *Proceedings of the 40th IEEE Conference on Decision and Control*, (Orlando, FL), December 2001.
- [5] J. Gravdahl, O. Egeland, and S. Vatland, "Active surge control of centrifugal compressors using drive torque," *Automatica*, vol. 38, no. 11, 2002.
- [6] E. Greitzer, "Surge and Rotating stall in axial flow compressors, Part I: Theoretical compression system model," *Journal of Engineering for Power*, vol. 98, pp. 190–198, 1976.
- [7] D. Fink, N. Cumpsty, and E. Greitzer, "Surge dynamics in a free-spool centrifugal compressor system," *Journal of Turbomachinery*, vol. 114, pp. 321–332, 1992.
- [8] J. Gravdahl and O. Egeland, "Centrifugal compressor surge and speed control," *IEEE Transactions on Control Systems Technology*, vol. 7, no. 5, 1999.
- [9] A. Robertson, *On Observer-Based Control of Nonlinear Systems*. PhD thesis, Lund Institute of Technology, 1999.
- [10] E. Panteley and A. Loria, "On global uniform asymptotic stability of nonlinear time-varying systems in cascade," *Systems and control letters*, vol. 33, pp. 131–138, 1998.
- [11] A. Loria, T. Fossen, and E. Panteley, "A separation principle for dynamic positioning of ships: Theoretical and experimental results," *IEEE Transactions on Control Systems Technology*, vol. 8, no. 2, pp. 332–343, 2000.
- [12] M. Seron and D. Hill, "Input-output and input-to-state stabilization of cascaded nonlinear systems," in *Proceeding of the 34th IEEE Conference on Decision and Control*, (New Orleans, La), pp. 4259–4264, 1995.

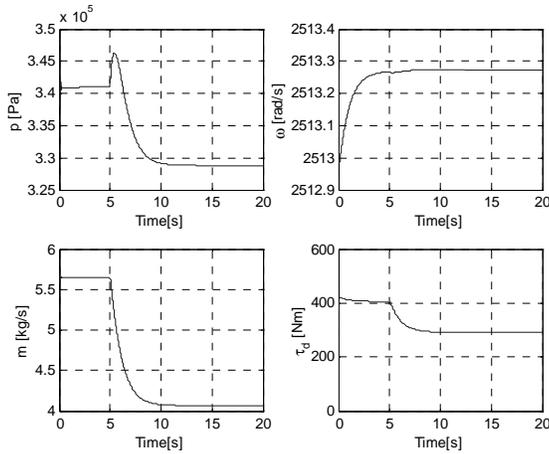


Figure 4: Active surge control using estimated mass flow

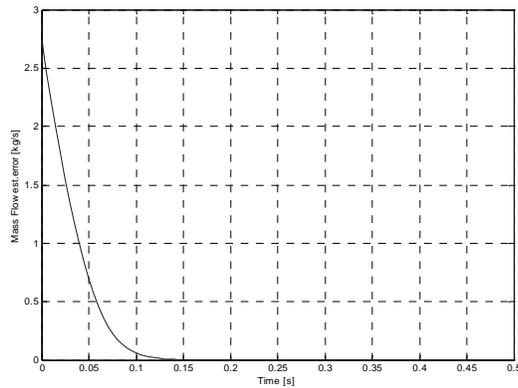


Figure 5: Mass flow estimation error

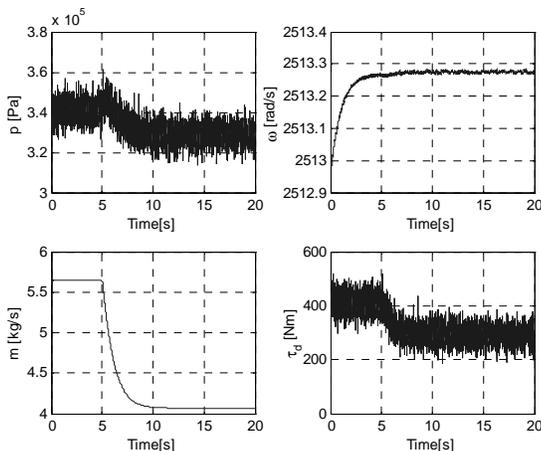


Figure 6: Active surge control using estimated mass flow and measurement noise